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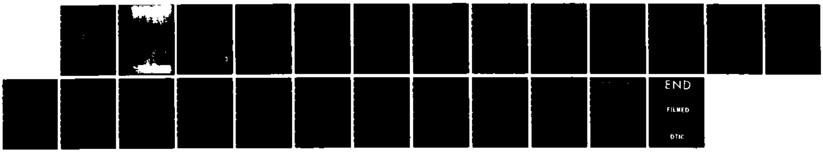

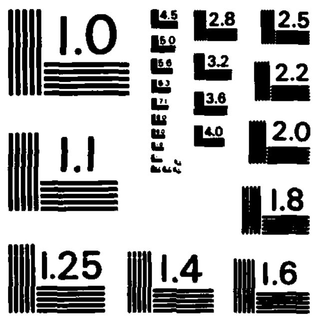

MICROCOPY RESOLUTION TEST CHART
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PREMATURELY TERMINATED SEQUENTIAL TESTS FOR MIL-STD 781C

by

C. Derman, Columbia University**
G.J. Lieberman, Stanford University*
Z. Schechner, Columbia University**

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1. Summary

Formulas and tables are provided making it possible for users of the truncated sequential probability ratio tests of MIL-STD 781C to make acceptance or rejection decisions when the test procedure is prematurely truncated.

2. Introduction

Users of the truncated sequential probability ratio tests given in MIL-STD 781C [2] are typically concerned with sampling a lot of items each of which is assumed to have identical exponential life distributions. A number n of items are kept on test; as soon as one fails, it is replaced by another. Let θ denote the mean time between failures. If $\theta = \theta_0$ the lot is considered acceptable; if $\theta = \theta_1$, $\theta_1 < \theta_0$, the lot is considered unacceptable. Let $N(t)$ denote the number of failures by time t where, throughout, t is measured in units of θ_1. The truncated MIL-STD 781C tests provide two boundaries, an upper boundary $A(t)$ and a lower boundary $B(t)$ and a truncation time T. If by time T, $N(t)$ crosses $A(t)$ before it crosses $B(t)$, then the hypothesis $\theta = \theta_1$ is accepted; if $N(t)$ crosses $B(t)$ before it crosses $A(t)$, or neither boundary is crossed by time $t = T$, then the hypothesis $\theta = \theta_0$ is accepted. A producer's risk α (probability of rejecting θ_0 when it is true) and a

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consumer's risk β (probability of accepting θ_0 when θ_1 is true) are provided for each of the eight tests in MIL-STD 781C.

Consider the following problem.[1] A lot is being tested using one of the eight tests of MIL-STD 781C. At some time t $(t < T)$ it becomes necessary to make a determination about the lot; i.e., to either accept or reject the lot. If $N(s)$ has crossed either $A(s)$ or $B(s)$ at some $s \leq t$, then the decision is dictated by the MIL-STD 781C test being used. However, if such is not the case, the user of MIL-STD 781C has no basis for making a decision. We present, here, a procedure that permits prematurely terminated MIL-STD 781C sequential testing and construct tables and decision rules that can be used to supplement MIL-STD 781C for use in such situations. Our calculations are based on the method presented by Kao, Kao, and Mogg [1].

3. Prematurely Terminated Sequential Tests (PTST)

For $t < T$, let $\bar{p}_{k,t}(\theta/\theta_1)$ denote the probability, when θ is the true mean time between failures, that $N(t) = k$ and $N(s)$ has not crossed either $A(s)$ or $B(s)$ for $s \leq t$. Each truncated MIL-STD 781C test has associated sets of values $\{u_i\}$ and $\{v_i\}$; u_i is the solution of $A(u_i) = i$ and v_i is the solution to $B(v_i) = i$ where the i's are integers. These numbers, called reject and accept numbers in MIL-STD 781C, are listed with each MIL-STD 781C test. They depend on the particular test used. Using the method in [1], it is possible to calculate, for each test,

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[1]The problem was presented by William Wallace of the Naval Electronics System Command at the Quality Control Workshop held in Washington, D.C. on on March 7-8, 1983.

associated sets of values $\{A_i(\theta/\theta_1)\}$ and $\{B_i(\theta/\theta_1)\}$; the range of indices, i, of $\{A_i(\theta/\theta_1)\}$ are the same as for $\{u_i\}$ and the range of indices, i, of $\{B_i(\theta/\theta_1)\}$ are the same as for $\{v_i\}$. The interpretation of $B_i(\theta/\theta_1)$ is that it is the probability, when θ is the true mean time between failures, of $N(t)$ crossing $B(t)$ at $t = v_i$ not having previously crossed either boundary ($N(t)$ can cross $B(t)$ only at points $\{v_i\}$). $A_i(\theta/\theta_1)$, also, has a boundary crossing probability interpretation; however, for our purposes this does not have to be made explicit. The equation for $\bar{p}_{k,t}(\theta/\theta_1)$ is given by

$$(1) \quad \bar{p}_{k,t}\left(\frac{\theta}{\theta_1}\right) = \frac{\exp\left[-\frac{\theta}{\theta_1}t\right]\left(\frac{\theta}{\theta_1}t\right)^k}{k!}$$

$$- \sum_{i=p}^{k} A_i\left(\frac{\theta}{\theta_1}\right) \exp\left[-\frac{\theta}{\theta_1}(t-u_i)\right] \frac{\left(\frac{\theta}{\theta_1}(t-u_i)\right)^{k-i}}{(k-i)!}$$

$$- \sum_{j=0}^{q'(t)} B_j\left(\frac{\theta}{\theta_1}\right) \exp\left[-\frac{\theta}{\theta_1}(t-v_j)\right] \frac{\left(\frac{\theta}{\theta_1}(t-v_j)\right)^{k-j}}{(k-j)!},$$

where p equals one plus the greatest integer less than $A(0)$ and $q'(t)$ is the greatest integer less than $B(t)$. Let c_t be a critical number at time t; i.e., if it should be necessary at time t to accept or reject $\theta = \theta_0$, then accept $\theta = \theta_0$ if $N(t) \leq c_t$. Then,

(2) $P\{\text{Accepting } \theta_0 \text{ by time } t|\theta\} = P\{\text{Accepting } \theta_0 \text{ prior to time } t|\theta)$

$$+ P\{\text{Accepting } \theta_0 \text{ at time } t|\theta\}$$

$$= \sum_{j=0}^{q'(t)} B_j(\theta/\theta_1) + \sum_{k=[q'(t)]+1}^{c_t} \bar{P}_{k,t}(\theta/\theta_1) \ ,$$

where, in the case $t < v_0$, the lower limit of summation, $[q'(t)] + 1$, is interpreted as zero.

Given c_t, a test based on the termination time t and the original truncated MIL-STD 781C test is defined. We refer to this as a prematurely terminated sequential test (PTST). The producer's risk $\alpha(c_t)$ is given by

(3) $$\alpha(c_t) = 1 - P\{\text{Accepting } \theta_0 \text{ by time } t|\theta_0\}$$

and the consumer's risk $\beta(c_t)$ is given by

(4) $$\beta(c_t) = P\{\text{Accepting } \theta_0 \text{ by time } t|\theta_1\} \ .$$

We stress that for (3) and (4) to be valid, the termination time t must be independent of $N(s)$, $s \leq t$. For otherwise, a value of t could be selected to favor one or the other hypothesis.

If, for each t, c_t is chosen such that $\alpha(c_t) \leq \alpha$, the producer's risk of the truncated MIL-STD 781C in use, then the producer's risk for the overall PTST procedure, allowing for any premature termination time t, which is independent of $N(s)$, $s \leq t$, to be less than or equal to α. Exact equality, if desired, can be achieved by a suitable use of randomization. In the same way the consumer's risk can be controlled. However, both risks cannot be simultaneously guaranteed.

There are two other problems that may be of interest. The user may be concerned with the conditional probability that he accepts θ_0 given that he has been forced to make a decision at time t and $N(s)$, $s \leq t$, has not crossed a boundary. This conditional probability is simply

$$\sum_{k=[q'(t)]+1}^{c_t} \bar{p}_{k,t}(\theta/\theta_1) / \sum_{B(t)<k<A(t)} \bar{p}_{k,t}(\theta/\theta_1) .$$

The user may also be interested in more flexibility in choosing, initially, a truncation time. The PTST provides the means for constructing additional tests with a greater variety of truncation times.

4. Tables of Risks for PTST

For each of the eight truncated sequential tests of MIL-STD 781C we calculate $\{A_i(\theta/\theta_1)\}$, $\{A_i(l)\}$, $\{B_i(\theta/\theta_1)\}$ *and* $\{B_i(l)\}$. *For* values of $t = 1/5\ T, \ldots, 4/5\ T$ we tabulate $\bar{p}_{k,t}(\theta_i/\theta_1)$ $(i = 0,1)$ for the relevant values of k. We define $c(t,\alpha)$ to be the smallest value of c_t such that $\alpha(c_t) \leq \alpha$. We find $c(t,\alpha)$ and calculate $\alpha(t) = \alpha(c(t,\alpha))$ and $\beta(t) = \beta(c(t,\alpha))$.

If $t \neq 1/5\ T, \ldots, T$, then one can approximate by using the largest multiple of $T/5$ less than t for his termination time or the formulas (1)-(4) can be used to derive the PTST for that particular time t.

5. Example

Suppose a shipment of electronic equipment is to be tested. The life of each piece of equipment is assumed to be exponential with mean μ. It

would be acceptable if $\mu = \mu_0 = 100$ hours and unacceptable if $\mu = \mu_1 = 67$ hours. It is decided to let $n = 10$ so that $\theta_0 = 10$ and $\theta_1 = 6.7$. Test Plan IC, which has an $\alpha = \beta = .10$ and a $T = 49.50$ measured in units of 6.7 hours, is to be used. Suppose the test is prematurely terminated at $t = 20$ (134 hours). Since this is not one of our values of T in our tabulation, we round down to $2/5(49.5)) = 19.8$. From the tables the critical number at 19.8 is 18. Thus, the test used is: Accept θ_0 if prior to $T = 19.8$, $N(s)$ crosses $B(s)$ before crossing $A(s)$ or if $N(19.8)) \leq 18$. The value of $\alpha(19.8) = .085$. The value of $\beta(19.8) = .388$.

If it is relevant to consider t as a premature termination necessitating an early decision given that no decision has been reached; the conditional probability that the hypothesis $\theta = \theta_0$ is rejected (when $\theta = \theta_0$) is

$$\sum_{k=19}^{21} \bar{P}_{k,19.8}(1.5) / \sum_{k=11}^{21} \bar{P}_{k,19.8}(1.5) = .0726 .$$

When $\theta = \theta_1$, the conditional probability that the hypothesis $\theta = \theta_0$ is accepted is

$$\sum_{k=11}^{18} \bar{P}_{k,19.8}(1) / \sum_{k=11}^{21} \bar{P}_{k,19.8}(1) = .7299 .$$

These conditional probabilities can be interpreted as the risks associated with only premature decisions (excluding those cases where decisions may have been reached using the MIL-STD procedure).

If, in fact, $\theta = 10$, the conditional probability that the hypothesis $\theta = \theta_0 = 10$ is rejected given $N(s)$ has not crossed either $A(s)$ or $B(s)$ prior to $t = 19.8$ is

$$\sum_{k=19}^{21} \bar{p}_{k,19.8}(1.5) / \sum_{k=11}^{21} \bar{p}_{k,19.8}(1.5) = .0726$$

and if, in fact $\theta = 6.7$, the conditional probability that the hypothesis $\theta = \theta_0 = 10$ is accepted given $N(s)$ has not crossed either $A(s)$ or $B(s)$ prior to $t = 19.8$ is

$$\sum_{k=11}^{18} \bar{p}_{k,19.8}(1) / \sum_{k=11}^{21} \bar{p}_{k,19.8}(1) = .7299 .$$

Acknowledgment: We are indebted to Larry Wein for the computer programming required for the calculations performed in this paper.

MIL-STD 781C TEST IC

	$\alpha = .10$, $\theta = \theta_0$		$\beta = .10$, $\theta_1 = .67\ \theta_0$	
i	$A_i(1.5)$	$B_i(1.5)$	$A_i(1)$	$B_i(1)$
0		0.0123		0.0014
1		0.0240		0.0027
2		0.0322		0.0036
3		0.0369		0.0041
4		0.0394		0.0044
5		0.0399		0.0044
6	0.0000	0.0391	0.0001	0.0043
7	0.0003	0.0384	0.0026	0.0042
8	0.0010	0.0369	0.0087	0.0041
9	0.0017	0.0354	0.0154	0.0039
10	0.0023	0.0335	0.0211	0.0037
11	0.0027	0.0319	0.0249	0.0035
12	0.0030	0.0300	0.0277	0.0033
13	0.0032	0.0285	0.0290	0.0032
14	0.0033	0.0267	0.0297	0.0030
15	0.0032	0.0253	0.0296	0.0028
16	0.0032	0.0237	0.0293	0.0026
17	0.0032	0.0221	0.0289	0.0024
18	0.0030	0.0210	0.0277	0.0023
19	0.0029	0.0197	0.0267	0.0022
20	0.0028	0.0186	0.0255	0.0021
21	0.0027	0.0174	0.0245	0.0019
22	0.0025	0.0165	0.0232	0.0018
23	0.0024	0.0155	0.0222	0.0017
24	0.0023	0.0146	0.0210	0.0016
25	0.0022	0.0137	0.0200	0.0015
26	0.0021	0.0130	0.0189	0.0014
27	0.0020	0.0121	0.0180	0.0013
28	0.0019	0.0113	0.0172	0.0013
29	0.0018	0.0108	0.0161	0.0012
30	0.0017	0.0101	0.0151	0.0011
31	0.0016	0.0096	0.0145	0.0011
32	0.0015	0.0090	0.0137	0.0010
33	0.0014	0.0085	0.0129	0.0009
34	0.0013	0.0079	0.0123	0.0009
35	0.0013	0.0075	0.0115	0.0008
36	0.0012	0.0128	0.0109	0.0019
37	0.0011	0.0195	0.0103	0.0044
38	0.0011	0.0216	0.0098	0.0072
39	0.0010	0.0207	0.0093	0.0104
40	0.0010	0.0180	0.0087	0.0136
41	0.0145		0.0164	

MIL-STD 781C TEST IC
(Continued)

t	k	$\bar{p}_{k,t}(1.5)$	$\bar{p}_{k,t}(1)$
9.90	3	0.0466	0.0058
	4	0.1005	0.0188
	5	0.1399	0.0392
	6	0.1556	0.0654
	7	0.1471	0.0927
	8	0.1214	0.1148
	9	0.0890	0.1261
	10	0.0584	0.1242
	11	0.0341	0.1087
	12	0.0167	0.0801
	13	0.0050	0.0360
19.80	11	0.0360	0.0042
	12	0.0720	0.0127
	13	0.0912	0.0241
	14	0.0943	0.0374
	15	0.0863	0.0514
	16	0.0721	0.0645
	17	0.0555	0.0744
	18	0.0392	0.0788
	19	0.0249	0.0750
	20	0.0133	0.0601
	21	0.0046	0.0312
29.70	19	0.0199	0.0022
	20	0.0420	0.0070
	21	0.0547	0.0137
	22	0.0576	0.0216
	23	0.0535	0.0301
	24	0.0454	0.0383
	25	0.0355	0.0450
	26	0.0256	0.0486
	27	0.0166	0.0475
	28	0.0092	0.0395
	29	0.0035	0.0227
39.60	28	0.0231	0.0036
	29	0.0327	0.0077
	30	0.0352	0.0125
	31	0.0333	0.0177
	32	0.0286	0.0228
	33	0.0227	0.0272
	34	0.0166	0.0298
	35	0.0110	0.0296
	36	0.0063	0.0254
	37	0.0026	0.0159
	38	0.0000	0.0002

MIL-STD 781C TEST IC
(Continued)

t	c(t,.1)	α(t)	β(t)
9.90	10	.073	.594
19.80	18	.085	.388
29.70	26	.089	.270
39.60	34	.090	.201

MIL-STD 781C TEST IIC

	$\alpha = .20$, $\theta = \theta_0$		$\beta = .20$, $\theta_1 = .67\ \theta_0$	
i	$A_i(1.5)$	$B_i(1.5)$	$A_i(1)$	$B_i(1)$
0		0.0612		0.0151
1		0.0763		0.0189
2		0.0746		0.0185
3	0.0006	0.0686	0.0018	0.0170
4	0.0139	0.0611	0.0433	0.0151
5	0.0199	0.0542	0.0622	0.0135
6	0.0209	0.0472	0.0649	0.0117
7	0.0195	0.0414	0.0604	0.0102
8	0.0174	0.0365	0.0541	0.0090
9	0.0156	0.0318	0.0483	0.0079
10	0.0137	0.0280	0.0426	0.0069
11	0.0122	0.0243	0.0378	0.0060
12	0.0107	0.0221	0.0332	0.0056
13	0.0095	0.0184	0.0295	0.0046
14	0.0083	0.0161	0.0259	0.0040
15	0.0074		0.0230	
16	0.0065		0.0202	
17	0.0058		0.0180	
18	0.0051		0.0159	
19	0.0167		0.0250	

MIL-STD 781C TEST IIC

(Continued)

t	k	$\bar{P}_{k,t}(1.5)$	$\bar{P}_{k,t}(1)$
4.38	1	0.1507	0.0525
	2	0.2295	0.1199
	3	0.2237	0.1754
	4	0.1613	0.1896
	5	0.0850	0.1499
	6	0.0201	0.0531
8.76	4	0.0741	0.0202
	5	0.1331	0.0545
	6	0.1451	0.0892
	7	0.1204	0.1110
	8	0.0780	0.1078
	9	0.0343	0.0712
13.14	8	0.0614	0.0197
	9	0.0888	0.0428
	10	0.0851	0.0614
	11	0.0637	0.0690
	12	0.0364	0.0592
	13	0.0116	0.0283
17.52	11	0.0252	0.0063
	12	0.0487	0.0184
	13	0.0555	0.0314
	14	0.0477	0.0405
	15	0.0324	0.0413
	16	0.0159	0.0303
	17	0.0022	0.0062

t	$c(t,.2)$	$\alpha(t)$	$\beta(t)$
4.38	4	.174	.553
8.76	8	.168	.452
13.14	12	.180	.372
17.52	16	.194	.312

MIL-STD 781C TEST IIIC

	$\alpha = .10$, $\theta = \theta_0$		$\beta = .10$, $\theta_1 = .5\ \theta_0$	
i	$A_i(2)$	$B_i(2)$	$A_i(1)$	$B_i(1)$
0		0.1108		0.0123
1		0.1217		0.0135
2		0.1090		0.0120
3	0.0050	0.0931	0.0284	0.0103
4	0.0155	0.0779	0.0876	0.0087
5	0.0167	0.0639	0.0940	0.0071
6	0.0147	0.0531	0.0829	0.0059
7	0.0124	0.0438	0.0703	0.0049
8	0.0105	0.0358	0.0591	0.0040
9	0.0088	0.0294	0.0494	0.0032
10	0.0072	0.0242	0.0409	0.0027
11	0.0061	0.0198	0.0342	0.0022
12	0.0051		0.0285	
13	0.0042		0.0236	
14	0.0035		0.0196	
15	0.0029		0.0164	

MIL-STD 781C TEST IIIC

(Continued)

t	k	$\bar{P}_{k,t}(2)$	$\bar{P}_{k,t}(1)$
4.12	0	0.1275	0.0162
	1	0.2626	0.0669
	2	0.2704	0.1379
	3	0.1848	0.1884
	4	0.0885	0.1805
	5	0.0202	0.0825
8.24	3	0.1092	0.0142
	4	0.1647	0.0428
	5	0.1477	0.0768
	6	0.0974	0.1012
	7	0.0465	0.0966
	8	0.0103	0.0428
12.36	6	0.0635	0.0084
	7	0.0930	0.0247
	8	0.0815	0.0432
	9	0.0528	0.0560
	10	0.0248	0.0525
	11	0.0052	0.0221
16.48	9	0.0359	0.0048
	10	0.0518	0.0140
	11	0.0448	0.0242
	12	0.0287	0.0311
	13	0.0133	0.0288
	14	0.0027	0.0116

t	$c(t,.1)$	$\alpha(t)$	$\beta(t)$
4.12	4	.066	.590
8.24	7	.093	.369
12.36	11	.103*	.271
16.48	14	.114*	.193

*Although $\alpha = .10$ is the nominal producer's risk in MIL-STD 781C IIIC, our method of calculation of the MIL STD producer's risk yields an α which exceeds our tabulated values.

MIL-STD 781C TEST IVC

$\alpha = .20$, $\theta = \theta_0$ | $\beta = .20$, $\theta_1 = .5\ \theta_0$

i	$A_i(2)$	$B_i(2)$	$A_i(1)$	$B_i(1)$
0		0.2466		0.0608
1		0.1732		0.0428
2	0.0432	0.1158	0.1217	0.0284
3	0.0513	0.0781	0.1451	0.0192
4	0.0380	0.0523	0.1077	0.0129
5	0.0276		0.0778	
6	0.0194		0.0548	
7	0.0137		0.0388	

t	k	$\bar{P}_{k,t}(2)$	$\bar{P}_{k,t}(1)$
1.95	0	0.3776	0.1426
	1	0.3678	0.2777
	2	0.1560	0.2356
3.90	1	0.1996	0.0569
	2	0.2403	0.1371
	3	0.1371	0.1563
	4	0.0245	0.0559
5.84	3	0.1364	0.0587
	4	0.1062	0.0915
	5	0.0365	0.0629
7.79	4	0.0688	0.0224
	5	0.0728	0.0474
	6	0.0362	0.0471
	7	0.0028	0.0074

t	c(t,.2)	$\alpha(t)$	$\beta(t)$
1.95	2	.099	.656
3.90	3	.176	.411
5.84	5	.185	.345
7.79	7	.206*	.276

*Although $\alpha = .20$ is the nominal producer's risk in MIL-STD 781C IVC, our method of calculation of the MIL STD producer's risk yields an α which exceeds out tabulated value.

MIL-STD 781C TEST VC

	$\alpha = .10$, $\theta = \theta_0$		$\beta = .10$, $\theta_1 = .33\ \theta_0$	
i	$A_i(3)$	$B_i(3)$	$A_i(1)$	$B_i(1)$
0		0.2865		0.0235
1		0.2066		0.0170
2	0.0149	0.1384	0.0919	0.0113
3	0.0275	0.0911	0.1689	0.0074
4	0.0200		0.1230	
5	0.0138		0.0845	
6	0.0094		0.0575	

t	k	$\bar{P}_{k,t}(3)$	$\bar{P}_{k,t}(1)$
2.07	0	0.5016	0.1262
	1	0.3461	0.2612
	2	0.1103	0.2498
4.14	1	0.3145	0.0597
	2	0.2329	0.1327
	3	0.0902	0.1541
	4	0.0072	0.0370
6.21	2	0.1831	0.0262
	3	0.1576	0.0678
	4	0.0708	0.0903
	5	0.0125	0.0483
8.28	3	0.1048	0.0113
	4	0.1042	0.0338
	5	0.0535	0.0521
	6	0.0136	0.0396

t	c(t,.1)	$\alpha(t)$	$\beta(t)$
2.07	2	.042	.637
4.14	3	.076	.370
6.21	4	.095	.226
8.28	6	.092	.189

MIL-STD 781C TEST VIC

	$\alpha = .20$, $\theta = \theta_0$		$\beta = .20$, $\theta_1 = .33\ \theta_0$	
i	$A_i(3)$	$B_i(3)$	$A_i(1)$	$B_i(1)$
0		0.4107		0.0693
1		0.2109		0.0355
2	0.0064		0.1452	

t	k	$\bar{P}_{k,t}(3)$	$\bar{P}_{k,t}(1)$
0.90	0	0.7408	0.4066
	1	0.2222	0.3659
	2	0.0280	0.1383
1.80	0	0.5488	0.1653
	1	0.3293	0.2975
	2	0.0948	0.2571
2.70	1	0.3618	0.1794
	2	0.1617	0.2406
3.60	1	0.2681	0.0730
	2	0.2002	0.1635

t	c(t,.2)	$\alpha(t)$	$\beta(t)$
0.90	1	.037	.772
1.80	1	.122	.463
2.70	2	.066	.489
3.60	2	.121	.306

MIL-STD 781C TEST VIIC

	$\alpha = .30$, $\theta = \theta_0$		$\beta = .30$, $\theta_1 = .67\ \theta_0$	
i	$A_i(1.5)$	$B_i(1.5)$	$A_i(1)$	$B_i(1)$
0		0.1225		0.0429
1		0.1140		0.0399
2		0.0948		0.0332
3	0.0398	0.0763	0.0893	0.0267
4	0.0425		0.0957	
5	0.0367		0.0826	

t	k	$\bar{P}_{k,t}(1.5)$	$\bar{P}_{k,t}(1)$
1.36	0	0.4039	0.2567
	1	0.3662	0.3491
	2	0.1660	0.2374
	3	0.0140	0.0299
2.72	0	0.1631	0.0659
	1	0.2958	0.1792
	2	0.2682	0.2437
	3	0.1475	0.2010
	4	0.0238	0.0488
4.08	1	0.1383	0.0533
	2	0.2310	0.1334
	3	0.2124	0.1840
	4	0.1244	0.1617
	5	0.0278	0.0542
5.44	2	0.1041	0.0382
	3	0.1792	0.0986
	4	0.1699	0.1403
	5	0.1046	0.1296

t	$c(t,.3)$	$\alpha(t)$	$\beta(t)$
1.36	1	.230	.606
2.72	2	.273	.489
4.08	3	.296	.414
5.44	5	.206	.489

MIL-STD 781C TEST VIIIC

	$\alpha = .30$, $\theta = \theta_0$		$\beta = .30$, $\theta_1 = .5\ \theta_0$	
i	$A_i(2)$	$B_i(2)$	$A_i(1)$	$B_i(1)$
0		0.4232		0.1791
1		0.1825		0.0775

t	k	$\bar{P}_{k,t}(2)$	$\bar{P}_{k,t}(1)$
0.90	0	0.6376	0.4066
	1	0.2869	0.3659
	2	0.0646	0.1647
1.80	1	0.3497	0.2843
	2	0.1643	0.2673
2.70	1	0.2229	0.1156
	2	0.2051	0.2127
3.60	2	0.1592	0.1053

t	c(t,.3)	$\alpha(t)$	$\beta(t)$
0.90	1	.075	.772
1.80	1	.227	.463
2.70	2	.149	.507
3.60	2	.235	.362

References

[1] Peicher Kao, Edward P.C. Kao, Jack M. Mogg, "A Simple Procedure for Computing Performance Characteristics of Truncated Sequential Tests with Exponential Lifetimes," Technometrics, Vol. 21, No. 2, May 1979.

[2] Military Standard, Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution (MIL-STD 781C), October 21, 1977, Dept. of Defense, U.S.A.

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truncated sequential tests
reject numbers
accept numbers

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Formulas and tables are provided making it possible for users of the truncated sequential probability ratio tests of MIL-STD 781C to make acceptance or rejection decisions when the test procedure is prematurely truncated.

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